2023-2024 Grand Challenge Award Final Report

Awardee: **Brandon A. Jones, Associate Professor**
Aerospace Engineering and Engineering Mechanics

Research Award Title: **Domain Splitting for Cislunar Uncertainty Propagation**

Research Summary

As humanity races to extend its footprint out to the Moon, the United States must act quickly to develop and implement new surveillance capabilities to maintain awareness of space objects in cislunar space. In near-Earth space, Earth's gravity dominates the dynamics, orbits are stable, and trajectory changes require large thrust maneuvers. That is not the case for the space between the Earth and Moon. The n-body dynamics can induce a sensitive dependence on initial conditions, i.e., chaos, and even small perturbations yield vastly different trajectories. This allows satellites to perform efficient trajectory changes and move with more freedom in the region. This also makes propagated trajectories sensitive to initial uncertainties, poorly characterized perturbations, and unknown maneuvers. Accurate prediction of a satellite's translation state and associated uncertainty is essential to tracking space objects. Understanding how we can do this tractably over a growing number of objects is essential to future situational awareness in these new regimes. The goal of this work is to develop methods of physics-informed domain splitting for trajectories in cislunar space consistent with the problem of uncertainty propagation.

This work leveraged a combination of local and global sensitivity measures to identify the directions of peak uncertainty growth for a given trajectory. Local measures consider solution sensitivity at a single point, but global measures consider a set of points weighted by our probabilistic knowledge of the position and velocity at an initial time. Existing approaches in the astrodynamics community use sensitivity indices based solely on (i) local measures or (ii) instantaneous information on the probability density (e.g., direction of largest variance). Instead, we developed methods that use the local dynamics, i.e., the flow, to identify an orthogonal basis where one vector corresponds to the direction of peak uncertainty, and then weighted samples or splitting directions based on the global sensitivity of uncertainty growth in that direction. This improves the efficiency of uncertainty propagation algorithms, which is quantified via the number of evaluation of the orbit propagator. These techniques were used in the context of uncertainty quantification via Gaussian mixtures and Polynomial Chaos Expansions (PCEs).

Gaussian Mixtures: To produce propagated uncertainty described as a mixture of Gaussian kernels, we defined a sampling distribution via local and global sensitivity indices. Generated samples were then weighted based on their initial probability density in a manner analogous to importance sampling. This approach ensured that samples were drawn from the regions of the initial distribution that accounted for the most growth in propagated uncertainty. The propagated particles were then used in a k-nearest neighbors approach to form a Gaussian mixture from the discrete samples. Performance of the new approach was quantified and compared to existing methods based on hypothesized measurements that would have failed the data association process (i.e., the tagging of observations to space objects) required in the tracking of space objects. The sample results in Figure 1 compare solutions when considering the global sensitivity indices (left) and only using local information (right). Only using local measures of sensitivity result in one tail of the propagated distribution being missed for a trajectory that passes close to the Moon. Using the global measures mitigate this issue and yield no missed cases.

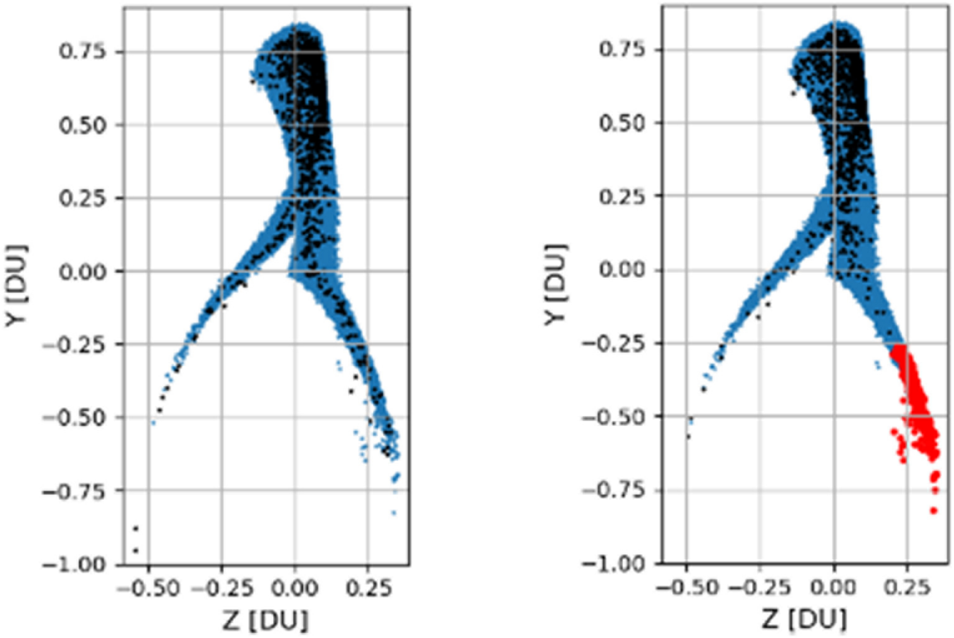

Figure 1: Proposed Monte Carlo Samples (blue), 1000 samples from the proposed sampling density (black), and any samples that fail a gating test for data association (red).

Polynomial Chaos Expansion: In this work, we combine local and global sensitivity indices to improve efficiency for propagating orbit uncertainty via multi-element PCEs. These multi-element approaches recursively split the input domain to achieve a prescribed accuracy in a surrogate representation of the propagated solution. Effectively, this breaks the larger problem into small, easier to solve sub-problems. We use the local sensitivity indices to develop an orthogonal transformation of the input variables to a basis that corresponds with the direction of peak uncertainty growth. We then prioritize directions for the multi-element splitting approach based on the Sobol indices, which are computed analytically using the PCE. For a near-Earth Molniya orbit, which is a known challenge case for orbit uncertainty propagation, this approach reduces the computation time of uncertainty propagation by an order of magnitude while improving accuracy of the polynomial chaos expansion. For the same lunar flyby case in Figure 1, this yields a 31% reduction in computation time and a factor of five improvement in accuracy (quantified as mean absolute error). Agreement in the distribution of samples produced via Monte Carlo propagation of the probability density function (black line) and samples from the polynomial chaos surrogate (histogram) is provided in Figure 2. Except for some cases in the tails of the distribution with low density (note the logarithmic scale), the new multi-element approach and a Monte Carlo analysis produce a close agreement in the solution.

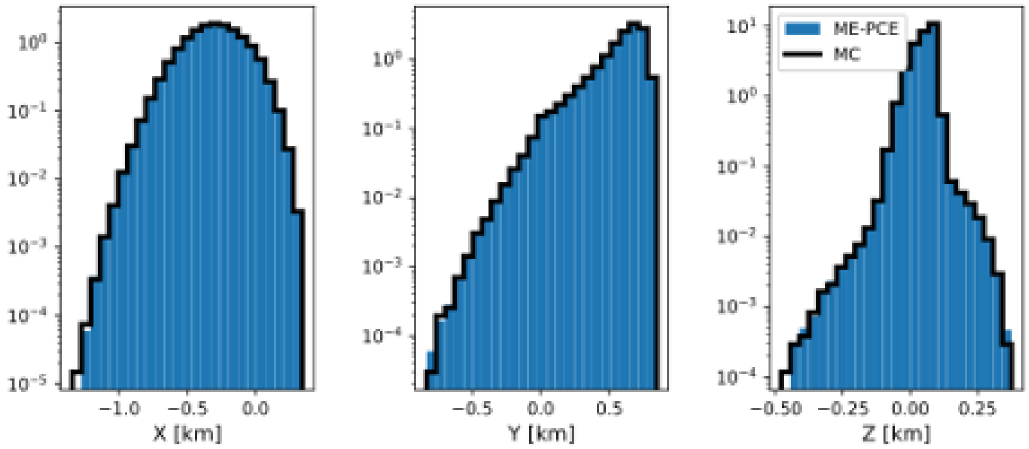

Figure 2: Normalized histograms for the new multi-element PCE domain segmentation approach in the lunar flyby scenario.

The work performed by this Grand Challenge Award has seeded one funded proposal and several other proposals under review at the time of this final report. Its importance and potential impact was recognized with one best-paper award at the IEEE Aerospace Conference, and a journal publication is currently being drafted based on this work.

Papers Published

- Jones, B.A., "Physics-Informed Domain Splitting for Orbit Uncertainty Propagation", *AIAA SCITECH 2024 Forum*, AIAA 2024-0203 (17 pages), Orlando, FL, Jan. 8-12, 2024, 10.2514/6.2024-0203.
- Jones, B.A., "Comparison of Sensitivity Metrics for Orbit Uncertainty Propagation in Cislunar Space", In *2024 IEEE Aerospace Conference*, pp. 1-14, Big Sky, MT, Mar. 2-9, 2024.

Presentations

- Jones, B.A., "Physics-Informed Domain Splitting for Orbit Uncertainty Propagation", *AIAA SCITECH 2024 Forum*, AIAA 2024-0203 (17 pages), Orlando, FL, Jan. 8-12, 2024, 10.2514/6.2024-0203.
- Jones, B.A., "Comparison of Sensitivity Metrics for Orbit Uncertainty Propagation in Cislunar Space", In *2024 IEEE Aerospace Conference*, pp. 1-14, Big Sky, MT, Mar. 2-9, 2024.
- Jones, B.A., "Applications of UQ Methods in Astrodynamics", University of Colorado Boulder, Boulder, CO, April 23, 2024.

Awards and Honors

- Best Paper in Track Award, Track 6: Remote Sensing, 2024 IEEE Aerospace Conference, 2024.
- *STARLIT: univerSiTy spAce stRategic technoLogy InitiaTive*, Sponsor: United States Space Force via the Universities Space Research Association (USRA), PI: Marcus Holzinger, Univ. of Colorado Boulder, coPI: Brandon Jones, share $352,544.