Second Semester Area A—CSE

CSE 386L, Mathematical Methods in Science and Engineering

Learning Objectives

1. Students will develop practical skills in the use of mathematics to solve problems. Topics covered should include advanced calculus (grad, curl, and div in curvilinear coordinates, and Gauss and Stokes Theorems), ODE's and dynamical systems, calculus of variations, and PDE's (Sturm-Liouville theory, separation of variables, Fourier transform, and Lax-Milgram theory).
2. Students will develop an appreciation of higher level mathematics (e.g., vector, metric, and Hilbert spaces) as a framework for understanding and solving practical problems.
3. Students will continue to learn graduate level mathematics by learning Hilbert space theory.

1. Multidimensional Real Analysis [1-2 weeks]

- 1.1. Directional and partial derivatives; multilinear functionals and differentials; Taylor's Theorem
- 1.2. Curvilinear systems of coordinates and grad, curl and div operators
- 1.3. Gauss and Stokes Theorems

2. Ordinary Differential Equations and Dynamical Systems [3-4 weeks]

- 2.1. Existence and uniqueness
- 2.2. Closed form solutions [through the HW]
- 2.3. Systems and eigenvalues
- 2.4. Elementary calculus of variations
- 2.5. Constrained optimization and Lagrange multipliers in $\mathbb{R}^n$
- 2.6. Lyapunov Stability

3. Introduction to Partial Differential Equations [4 weeks]

- 3.1. Classification and general behavior of PDEs
- 3.2. Method of characteristics for first order hyperbolic PDE's
- 3.3. Elementary Sturm-Liouville theory
- 3.4. Separation of variables applied to elliptic, parabolic, and hyperbolic PDEs
- 3.5. Discussion of existence, uniqueness, and singular behavior

4. Hilbert spaces and duality [3 weeks]

- 4.1. Inner product and L^2-spaces
- 4.2. Orthogonality: orthogonal projections and orthonormal bases
- 4.3. Fourier series and the Fourier transform
- 4.4. Riesz Representation Theorem
- 4.5. Closed Range Theorem for a continuous operator

5. Elliptic Partial Differential Equations [2 weeks]

- 5.1. Applications of the Fourier transform
- 5.2. Green's functions
- 5.3. Variational formulations in H^1
- 5.4. Lax-Milgram and Babuška-Nečas Theorems

Textbooks:

1. *Applied Functional Analysis*, 3rd Ed., J. T. Oden and L. Demkowicz, CRC Press, 2018.
2. *Ordinary and Partial Differential Equations*, J. W. Cain and A. M. Reynolds, Creative Commons, Virginia Commonwealth University, Richmond, 2010.